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Mark Scheme (Results)

Summer 2013

GCE Decision Mathematics 2 (6690/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes:

- bod – benefit of doubt
- ft – follow through
- the symbol $\surd$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- * The answer is printed on the paper
- ⌐ The second mark is dependent on gaining the first mark

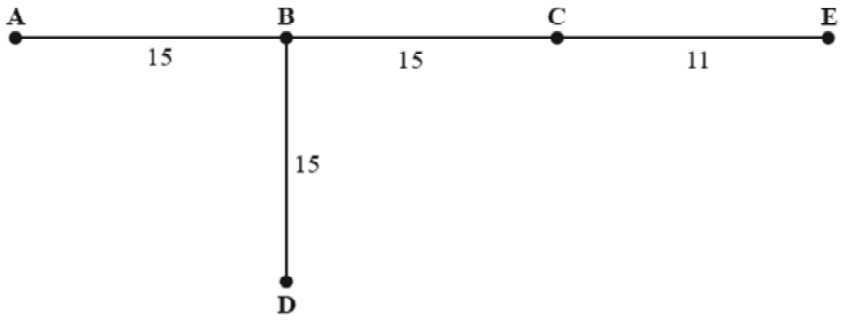
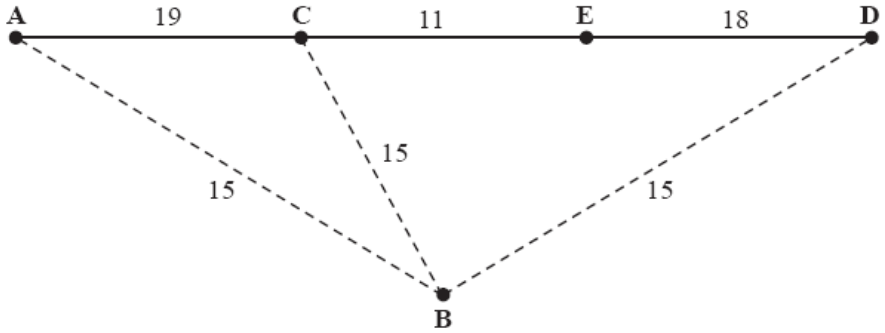
4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

8. In some instances, the mark distributions (e.g. M1, B1 and A1) printed on the candidate's response may differ from the final mark scheme.

Question Number	Scheme	Marks
1(a)	 e.g. starting from A: AB, BD, BC, CE or AB, BC, CE, BD	M1A1 (2)
(b)	2 x 56 = 112	B1 (1)
(c)	A B C E D A = 84 and A B D E C A = 78 15 15 11 18 25 15 15 18 11 19	M1 A1 A1 (3)
(d)	78 is the better upper bound	B1ft (1)
(e)	 Lower bound = 48 + 15 + 15 = 78	1M1A1 2M1A1 (4)

Question Number	Scheme	Marks
(f)	The route is ABDECA (The optimal route length is 78, since upper bound = lower bound) a1M1 First three arcs (or all 5 nodes / or numbers across the top of the matrix) selected correctly (may start from any node). Award M1 only for a correct tree with no working. a1A1 CAO (order of arc selection clear) b1B1 112 CAO c1M1 Nearest Neighbour either A-B-C-E-D- or A-B-D-E-C- (condone lack of return to start). Accept 12354 or 12534 across the top of the matrix. c1A1 1 route and length CAO (Do not ISW if route length is doubled) c2A1 both routes and lengths CAO (Do not ISW if route lengths are doubled) d1B1ft their stated shortest (must be a number) e1M1 Finding correct RMST (maybe implicit) 48 sufficient, or correct numbers. 3 arcs. e1A1 CAO; tree **or** 48 **or** 11 + 18 + 19 seen. e2M1 Adding 2 least arcs to B; 15 and 15 **or** two out of BA, BC or BD **or** 30 only e2A1 CAO 78 f1B1 CAO, accept any start point for the correct tour, but must return to start. Dependent on their answer to part (d) = their answer to part (e).	B1 (1) **Total 12**

Question Number	Scheme	Marks
2(a)		

	1	2	3	Supply
A	18			18
B	9	5		14
C		13	8	21
D			12	12
Demand	27	18	20	65

B1
(1)

(b)

	1	2	3
A	18		
B	9-θ	5+θ	
C		13-θ	8+θ
D	θ		12-θ

giving

	1	2	3
A	18		
B		14	
C		4	17
D	9		3

M1A1
(2)

(c)

Shadow costs		10	22	19
		1	2	3
0	A	X	-11	1
-15	B	20	X	9
-7	C	21	X	X
-1	D	X	0	X

1M1A1

	1	2	3
A	18-θ	θ	
B		14	
C		4-θ	17+θ
D	9+θ		3-θ

giving

	1	2	3
A	15	3	
B		14	
C		1	20
D	12		

2M1A1

(θ = 3) entering cell A2, exiting cell D3

(4)

Question Number	Scheme	Marks
(d)		

Shadow costs		10	11	8
		1	2	3
0	A	X	X	12
-4	B	9	X	9
4	C	10	X	X
-1	D	X	11	11

M1 A1

Optimal since no negative improvement indices.

A1
(3)

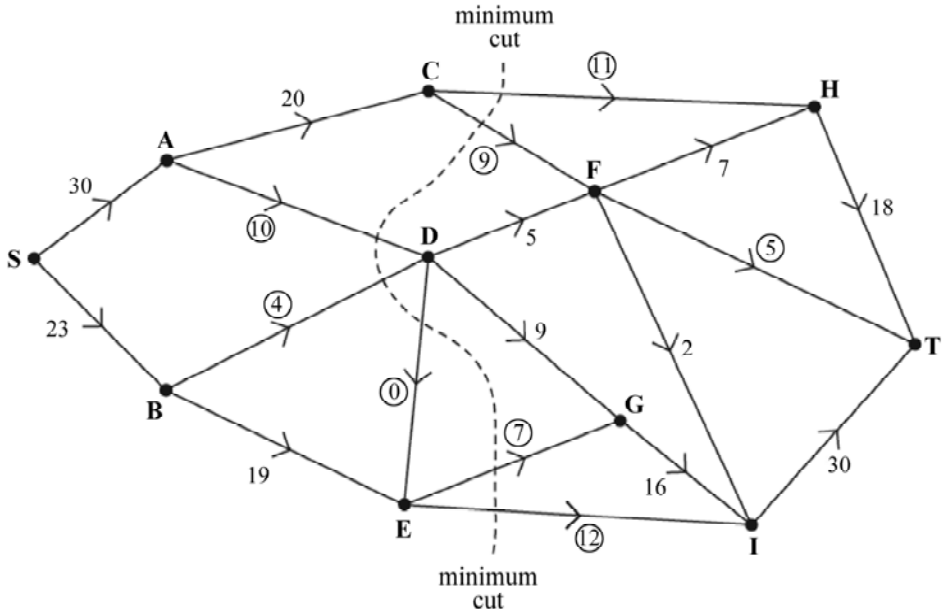
a1B1 CAO

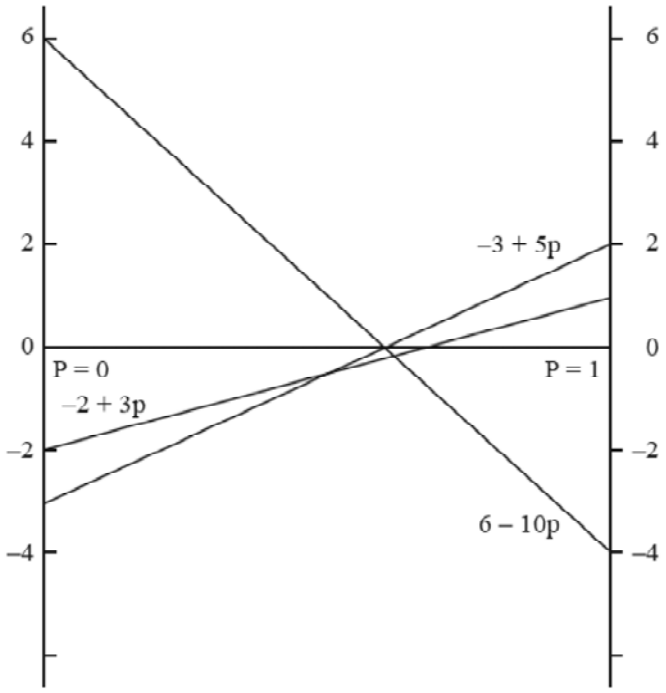
b1M1 A valid route, only one empty square, D1, used, θ's balance.
b1A1 Correct route, up to an improved solution (six numbers no zeros)

c1M1 Finding 7 shadow costs and 6 Improvement indices
c1A1 Shadow costs [Alt: A(10), B(-5), C(3), D(9), 1(0), 2(12), 3(9)] and improvement indices CAO
c2M1 A valid route, their most negative II chosen, only one empty square used, θ's balance.
c2A1 CSO (entering A2, and exiting D3 stated)

d1M1 Finding 7 shadow costs **and** all 6 IIs **or** at least1 negative II found.
d1A1 CAO for the shadow costs [Alt: A(10), B(6), C(14), D(9), 1(0), 2(1), 3(-2)] and 6 positive IIs
d2A1 CSO (for part (d)) + optimal.

Total 10

Question Number	Scheme	Marks
3(a)	Initial flow = 44	B1
(b)	Value of cut = 12+7+4+10+2+5+31 = 71	B1 (2)
(c)	e.g.SACFHT – 3; SADGIT – 4; SBEDFHT – 2 e.g. SACFHT – 3; SADFHT – 2; SADGIT – 2; SBEDGIT - 2	M1A1;A1; A1
(d)	e.g.  	(4) M1A1 (2)
(e)	Maximum flow=minimum cut e.g. cut through CH, CF, AD, BD, DE, EG and EI	DM1 A1 (2)
	a1B1 CAO b1B1 CAO c1M1 One valid flow augmenting route found and a value stated. c1A1 Flow increased by at least 2 c2A1 A second correct flow route (and value at least 2) correct c3A1 CSO Flow increased by 9 and no more. d1M1 Consistent flow pattern > 50 (check each node, must have exactly 1 number per arc) d1A1 CAO, showing flow of 53, must follow from their routes. e1DM1 Must have attempted (d) and made an attempt at a cut. e1A1 cut correct – may be drawn. Refer to max flow-min cut theorem all four words (alternative cut: CH, CF, AD, BD, BE). Guidance for 3(c) SA +7 SB +2 AC +3 AD +4 BD none BE + 2 ED + 2 CH none CF +3 EG none EI none (DF+2 DG+2 FH +5 FT none FI none GI +4 HT +5 IT +4)	**Total 10**

Question Number	Scheme	Marks
4(a)	$\begin{bmatrix} 4 & -6 \\ -2 & 3 \\ -1 & 2 \end{bmatrix}$ column 2 dominates column 1	B1 (1)
(b)	$\begin{bmatrix} -4 & 2 & 1 \\ 6 & -3 & -2 \end{bmatrix}$	B1 B1 (2)
(c)	Let p = probability that B plays new row 1 If A plays 1: B's expected winnings = -4p + 6(1- p) = 6 – 10p If A plays 2: B's expected winnings = 2p – 3(1 - p) = – 3 + 5p If A plays 3: B's expected winnings = p – 2(1 - p) = – 2 + 3p	1M1A1 (2)
		B2, 1ft, 0 (2)
	$6-10p=-2+3p$ $8=13p$ $p=\frac{8}{13}$	2M1 A1 (2)
	B should play 1: never, play 2 with probability $\frac{8}{13}$ and play 3 with probability $\frac{5}{13}$ The value of the game is $-\frac{2}{13}$ to B	B1 B1 (2) **Total 11**

Question Number	Scheme	Marks
	a1B1 CAO (accept reduced matrix or 'column 2 dominates column 1' or column crossed out). Allow recover in part (b) b1B1 either 3×2 matrix with correct values (including signs) or 2×3 matrix with correct values (condone incorrect signs) b2B1 CAO c1M1 Setting up three probability expressions, implicit definition of 'p'. c1A1 CAO (condone incorrect simplification) c1B1ft Attempt at three lines (correct gradients and intersection with 'axes'), accept $p > 1$ or $p < 0$ here. Must be functions of p. c2B1 CAO $0 \leq p \leq 1$, scale clear (or 1 line = 1), condone lack of labels. Rulers used. c2M1 Finding their correct optimal point, must have three lines and set up an equation to find $0 \leq p \leq 1$. Dependent on first B mark in part (c). Must have three intersection points. Solving all three simultaneous equations only is M0. c2A1 CSO c3B1 All three options listed must ft from their p, check page 1 for B should never play 1. $0 \leq$ probabilities ≤ 1. c4B1 -2/13 CAO (accept awrt 0.154) **SC1**: If column 2 deleted in (a) candidates can earn a **maximum** of (a) B0 (b) B1 B0 (c) M1 A0 B1 B0 M1 A0 B1 B1 (max. of 6) – the final B mark is for the value of the game being -4/3 **SC2**: If column 3 is deleted in (a) candidates can earn a **maximum** of (a) B0 (b) B1 B0 (c) M1 A0 B1 B0 M0 A0 B0 B0	

Question Number	Scheme	Marks
5(a)	Variable z was increased first, since it has become a basic variable.	B1
(b)	<table><tr><td>b.v</td><td>x</td><td>y</td><td>z</td><td>r</td><td>s</td><td>t</td><td>value</td></tr><tr><td>r</td><td>-1</td><td>$\boxed{2}$</td><td>0</td><td>1</td><td>0</td><td>1</td><td>8</td></tr><tr><td>s</td><td>-1</td><td>3</td><td>0</td><td>0</td><td>1</td><td>1</td><td>22</td></tr><tr><td>z</td><td>-2</td><td>1</td><td>1</td><td>0</td><td>0</td><td>1</td><td>11</td></tr><tr><td>P</td><td>2</td><td>-5</td><td>0</td><td>0</td><td>0</td><td>$\frac{1}{2}$</td><td>15</td></tr></table>	
	<table><tr><td>b.v</td><td>X</td><td>y</td><td>z</td><td>r</td><td>s</td><td>t</td><td>value</td><td>row ops</td></tr><tr><td>y</td><td>$-\frac{1}{2}$</td><td>1</td><td>0</td><td>$\frac{1}{2}$</td><td>0</td><td>$\frac{1}{2}$</td><td>4</td><td>$R_1 \div 2$</td></tr><tr><td>s</td><td>$\frac{1}{2}$</td><td>0</td><td>0</td><td>$-\frac{3}{2}$</td><td>1</td><td>$-\frac{1}{2}$</td><td>10</td><td>$R_2 - 3R_1$</td></tr><tr><td>z</td><td>$-\frac{3}{2}$</td><td>0</td><td>1</td><td>$-\frac{1}{2}$</td><td>0</td><td>$\frac{1}{2}$</td><td>7</td><td>$R_3 - R_1$</td></tr><tr><td>P</td><td>$-\frac{1}{2}$</td><td>0</td><td>0</td><td>$\frac{5}{2}$</td><td>0</td><td>3</td><td>35</td><td>$R_4 + 5R_1$</td></tr></table>	1M1A1 2M1A1 (4)
	<table><tr><td>b.v</td><td>X</td><td>y</td><td>z</td><td>r</td><td>s</td><td>t</td><td>value</td><td>row ops</td></tr><tr><td>y</td><td>0</td><td>1</td><td>0</td><td>-1</td><td>1</td><td>0</td><td>14</td><td>$R_1 + \frac{1}{2}R_2$</td></tr><tr><td>x</td><td>1</td><td>0</td><td>0</td><td>-3</td><td>2</td><td>-1</td><td>20</td><td>$R_2 \div \frac{1}{2}$</td></tr><tr><td>z</td><td>0</td><td>0</td><td>1</td><td>-5</td><td>3</td><td>-1</td><td>37</td><td>$R_3 + \frac{3}{2}R_2$</td></tr><tr><td>P</td><td>0</td><td>0</td><td>0</td><td>1</td><td>1</td><td>$\frac{5}{2}$</td><td>45</td><td>$R_4 + \frac{1}{2}R_2$</td></tr></table>	3M1A1ft 4M1A1 (4)
(c)	P =45; x = 20; y = 14; z = 37; r = s = t = 0. a1B1 Identifies *z*, refers to basic variable. b1M1 Correct pivot located, attempt to divide row. If choosing negative pivot M0M0. b1A1 CAO pivot row correct including change of b.v. b2M1 (ft) Correct row operations used at least once, column *x*, *r*, *t* or value correct. b2A1 CAO including row operations b3M1 Their correct pivot located, attempt to divide row. If choosing negative pivot M0M0. b3A1ft pivot row correct including change of b.v. b4M1 (ft) Correct row operations used at least once, column *r*, *s*, *t* or value correct. b4A1 CAO c1M1 Their correct values stated for at least *P, x, y, z* from their 'optimal' iteration. No negatives. Two M marks in part (b) must have been awarded c1A1 CAO for all 7 values.	M1 A1 (2) **Total 11**

Question Number	Scheme	Marks
6 (a)	Since maximising subtract all elements from some n≥257, say 260. $\begin{bmatrix} 9 & 17 & 3 \\ 16 & 13 & 5 \\ 11 & 8 & 14 \end{bmatrix}$ $\left(n=257 \begin{bmatrix} 6 & 14 & 0 \\ 13 & 10 & 2 \\ 8 & 5 & 11 \end{bmatrix}, \; n=258 \begin{bmatrix} 7 & 15 & 1 \\ 14 & 11 & 3 \\ 9 & 6 & 12 \end{bmatrix} \right)$	1B1 (1)
(b)	$x_{ij} = \begin{cases} 1 & \text{if worker } i \text{ does task } j \\ 0 & \text{otherwise} \end{cases}$	1B1
	Where x_{ij} indicates worker i being assigned to task j $i \in \{H, K, J\}$ and $j \in \{1, 2, 3\}$	2B1 (2)
	E.g. Minimise $P = 9x_{H1} + 17x_{H2} + 3x_{H3} + 16x_{J1} + 13x_{J2} + 5x_{J3} + 11x_{K1} + 8x_{K2} + 14x_{K3}$ $(P = 6x_{H1} + 14x_{H2} + 13x_{J1} + 10x_{J2} + 2x_{J3} + 8x_{K1} + 5x_{K2} + 11x_{K3})$ $(P = 7x_{H1} + 15x_{H2} + x_{H3} + 14x_{J1} + 11x_{J2} + 3x_{J3} + 9x_{K1} + 6x_{K2} + 12x_{K3})$	3B1 4B1
	OR maximise P = $251x_{H1} + 243x_{H2} + 257x_{H3} + 244x_{J1} + 247x_{J2} + 255x_{J3} + 249x_{K1} + 252x_{K2} + 246x_{K3}$	(2)
	Subject to: $x_{H1} + x_{H2} + x_{H3} = 1$ or $\sum x_{Hj} = 1$ $x_{J1} + x_{J2} + x_{J3} = 1$ or $\sum x_{Jj} = 1$ $x_{K1} + x_{K2} + x_{K3} = 1$ or $\sum x_{Kj} = 1$ $x_{H1} + x_{J1} + x_{K1} = 1$ or $\sum x_{i1} = 1$ $x_{H2} + x_{J2} + x_{K2} = 1$ or $\sum x_{i2} = 1$ $x_{H3} + x_{J3} + x_{K3} = 1$ or $\sum x_{i3} = 1$	M1 1A1 2A1 (3)
	a1B1 CAO (o.e.) b1B1 possible values of x_{ij} defined b2B1 Defining x_{ij} including the set of values for i and j b3B1 Objective function b4B1 Minimise/Maximise but consistent with objective function b1M1 Three equations, unit coefficients, =1 b1A1 Any three equations CAO (condone inconsistent notation)	**Total 8**

Question Number	Scheme	Marks
	b2A1 All six equations CAO (consistent notation required)	

Question Number	Scheme	Marks
7		

Stage	State	Action	Destination	Value
end	4	Sell	-	1*
	3	Sell	-	2*
	2	Sell	-	4*
	1	Sell	-	6*
4	3	K	4	1 + 2 – 3 = 0
		R	1	6 + 11 – 9 = 8*
	2	K	3	2 + 5 – 2 = 5
		R	1	6 + 11- 8 = 9*
	1	K	2	4 + 8 – 1 = 11*
		R	1	6 + 11 – 7 = 10
3	2	K	3	8 + 5 – 2 = 11
		R	1	11 + 11 – 8 = 14*
	1	K	2	9 + 8 – 1 = 16*
		R	1	11 + 11 – 7 = 15
2	1	K	2	14 + 8 – 1 = 21*
		R	1	16 + 11 – 7 = 20
1	new	K	1	21 + 11 = 32*

1M1A1A1

2M1 A1

3M1A1ft

A1

4M1 A1ft

A1

The actions Nigel should take are:
Keep, Keep, Replace, Keep in years 1, 2, 3 and 4 respectively — B1
His income will be £32 000. — B1ft

Total 13

1M1 At least 3 columns in Stage 4 completed, something in each cell.
1A1 For stage 4 at least two columns of state, action, destination entries correct
2A1 Two rows in Stage 4 CAO. Penalise * errors only twice in the question on the first occurrences
All future M marks must bring all optimal results from previous stage into current stage at least once (or three out of four previous results correct).
2M1 All four rows in stage 4 completed. Bod if something in each cell.
3A1 CAO. Stage 4 correct. (Penalise * errors only twice in the question).
3M1 Stage 3 completed. Bod if something in each cell.
4A1ft Any state correct (Penalise * errors only twice in the question).
5A1 CAO Both states correct. (Penalise * errors only twice in the question).
4M1 Stage 2 and 1 completed. Bod if something in each cell.
6A1ft CAO Stage 2 correct. (Penalise * errors only twice in the question).
7A1 CAO Stage 1 correct.
1B1 Actions correct. Must have earned all previous M marks
2B1ft Income correct for their table. Must have earned all previous M marks.
Penalise extra rows for stage 4 with the 3rd A mark, stage 3 with the 5th A mark and stage 2 with the 6th A mark.

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